Indefinite Integration

Question1

The value of $\int \frac{dx}{(x+1)(x+2)}$ is

KCET 2025

Options:

A. $\log\left|\frac{x-1}{x+2}\right| + c$

B. $\log\left|\frac{x-1}{x-2}\right| + c$

C. $\log\left|\frac{x+2}{x+1}\right| + c$

D. $\log\left|\frac{x+1}{x+2}\right| + c$

Answer: D

Solution:

$$\begin{aligned}\int \frac{dx}{(x+1)(x+2)} &= \int \frac{(x+2)-(x+1)}{(x+1)(x+2)} dx \\ &= \int \frac{1}{x+1} dx - \int \frac{1}{x+2} dx \\ &= \log\left|\frac{x+1}{x+2}\right| + c\end{aligned}$$

Question2

$\int \frac{dx}{x^2(x^4+1)^{3/4}}$ equals

KCET 2025

Options:

A. $\left(\frac{x^4+1}{x^4}\right)^{\frac{1}{4}} + c$

B. $(x^4+1)^{\frac{1}{4}}+c$

C. $-(\mathrm{x}^4+1)^{\frac{1}{4}}+\mathrm{c}$

D. $-\left(\frac{x^4+1}{x^4}\right)^{\frac{1}{4}}+c$

Answer: D

Solution:

$$\int \frac{\mathrm{dx}}{\mathrm{x}^2(\mathrm{x}^4+1)^{3/4}}$$
$$=\int \frac{\mathrm{dx}}{\mathrm{x}^5\left(1+\frac{1}{\mathrm{X}^4}\right)^{3/4}}$$

$$1+\frac{1}{x^4}=T$$
$$-4x^{-5}dx=dt$$

$$=-\frac{1}{4}\int \mathrm{t}^{-3/4}\mathrm{dt}=-\frac{1}{4}\frac{\mathrm{t}^{1/4}}{\frac{1}{4}}+\mathrm{c}=-\left(1+\frac{1}{\mathrm{x}^4}\right)^{1/4}$$

Question3

$$\int \frac{\sin x}{3+4\cos^2 x}dx$$

KCET 2024

Options:

A. $-\frac{1}{2\sqrt{3}}\tan^{-1}\left(\frac{2\cos x}{\sqrt{3}}\right)+C$

B. $\frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{\cos x}{3}\right)+C$

C. $\frac{1}{2\sqrt{3}}\tan^{-1}\left(\frac{\cos x}{3}\right)+C$

D. $-\frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{2\cos x}{3}\right)+C$

Answer: A

Solution:

$$\because I=\int \frac{\sin x}{3+4\cos^2 x}dx$$
$$=\frac{\sin x}{3\left[1+\left(\frac{2\cos x}{\sqrt{3}}\right)^2\right]^2}dx$$

put $t=\frac{2\cos x}{\sqrt{3}}$

$$dt=\frac{-2\sin x}{\sqrt{3}}dx$$
$$I=\int -\frac{\sqrt{3}}{2\left(3t^2+3\right)}dt$$
$$=-\frac{1}{2\sqrt{3}}\int \frac{1}{1+t^2}dt$$
$$=-\frac{1}{2\sqrt{3}}\tan^{-1}(t)+C$$
$$=\frac{-1}{2\sqrt{3}}\tan^{-1}\left(\frac{2\cos x}{\sqrt{3}}\right)+C$$

Question4

$\int \frac{1}{x[6(\log x)^2+7\log x+2]}dx$ is

KCET 2024

Options:

A. $\frac{1}{2}\log\left|\frac{2\log x+1}{3\log x+2}\right|+C$

B. $\log\left|\frac{2\log x+1}{3\log x+2}\right|+c$

C. $\log\left|\frac{3\log x+2}{2\log x+1}\right|+C$

D. $\frac{1}{2}\log\left|\frac{3\log x+2}{2\log x+1}\right|+C$

Answer: B

Solution:

$$I=\int \frac{1}{x\left[6(\log x)^2+7\log x+2\right]}dx$$
$$=\int \frac{dt}{6t^2+7t+2}\left[\begin{array}{l}\because \text{ put } t=\log x\\ dt=\frac{1}{x}dx\end{array}\right]$$
$$=\int \frac{dt}{(3t+2)(2t+1)}$$

Now, $\frac{1}{(3t+2)(2t+1)}=\frac{A}{(3t+2)}+\frac{B}{(2t+1)}$

$$=\frac{A(2t+1)+B(3t+2)}{(3t+2)(2t+1)}$$
$$\Rightarrow \quad 1=A(2t+1)+B(3t+2)$$

Now, on putting $t=\frac{-1}{2}$

$$\Rightarrow \quad 1=0+B\left[3\left(-\frac{1}{2}\right)+2\right]$$
$$\Rightarrow \quad B=2$$

and put $t=\frac{-2}{3}$

$$\Rightarrow \quad 1=A\left(2\times\left(-\frac{2}{3}\right)+1\right)+0\Rightarrow 1=A\left(-\frac{4}{3}+1\right)$$
$$\Rightarrow \quad A=-3$$
$$\therefore \quad I=\int\left(\frac{-3}{3t+2}+\frac{2}{2t+1}\right)dt$$
$$=\frac{-3\log|3t+2|}{3}+2\frac{\log|2t+1|}{2}+C$$
$$=-\log|3t+2|+\log|2t+1|+C$$
$$=\log\left|\frac{2t+1}{3t+2}\right|+C$$
$$=\log\left|\frac{2\log x+1}{3\log x+2}\right|+C \quad [\because t=\log x]$$

Question5

$\int \frac{\sin\frac{3x}{2}}{\sin\frac{x}{2}}dx$ **is**

KCET 2024

Options:

A. $2x+\sin x+2\sin 2x+C$

B. $x+2\sin x+2\sin 2x+C$

C. $x+2\sin x+\sin 2x+C$

D. $2x+\sin x+\sin 2x+C$

Answer: C

Solution:

$I=\int \frac{\sin\left(\frac{5x}{2}\right)}{\sin\left(\frac{x}{2}\right)} dx$

$$
\begin{aligned}
& =\int \frac{2 \sin \left(\frac{5x}{2}\right) \cos \left(\frac{x}{2}\right)}{\sin x} dx=\int \frac{\sin 3x+\sin 2x}{\sin x} dx \\
& =\int \frac{\left(3 \sin x-4 \sin ^3 x\right)+2 \sin x \cos x}{\sin x} dx \\
& =\int\left(3-4 \sin ^2 x+2 \cos x\right) dx \\
& =\int(3-2(1-\cos 2x)+2 \cos x) dx \\
& =\int(3-2+2 \cos 2x+2 \cos x) dx \\
& =\int(1+2 \cos 2x+2 \cos x) dx \\
& =x+\sin 2x+2 \sin x+C
\end{aligned}
$$

Question6

$\int \sqrt{\operatorname{cosec} x-\sin x} dx$ **is equals to**

KCET 2023

Options:

A. $\frac{\sqrt{\sin x}}{2}+C$

B. $2\sqrt{\sin x}+C$

C. $\frac{2}{\sqrt{\sin x}}+C$

D. $\sqrt{\sin x}+C$

Answer: B

Solution:

$$\text{Let} \quad I = \int \sqrt{\operatorname{cosec} x - \sin x} dx$$

$$I = \int \frac{\sqrt{1 - \sin^2 x}}{\sqrt{\sin x}} dx$$

$$= \int \frac{\cos x}{\sqrt{\sin x}} dx$$

Put, $\sin x - k$

$\cos x dx = dk$

$$I = \int \frac{dk}{\sqrt{k}} = \frac{k^{\frac{-1}{2}} + 1}{\frac{-1}{2} + 1} + C$$

$$= \frac{k^{\frac{1}{2}}}{\frac{1}{2}} + C = 2\sqrt{k} + C$$

$$I = 2\sqrt{\sin x} + C \quad [\because k = \sin x]$$

Question7

$\int \sqrt{5 - 2x + x^2} dx$ is equals to

KCET 2023

Options:

A. $\frac{x}{2} \sqrt{5 - 2x + x^2} + 4 \log \mid (x + 1) + \sqrt{x^2 - 2x + 5} \mid + C$

B. $\frac{x-1}{2} \sqrt{5 + 2x + x^2} + 2 \log \mid (x - 1) + \sqrt{5 + 2x + x^2} \mid + C$

C. $\frac{x-1}{2} \sqrt{5 - 2x + x^2} + 2 \log \mid (x - 1) + \sqrt{5 - 2x + x^2} \mid + C$

D. $\frac{x-1}{2} \sqrt{5 - 2x + x^2} + 2 \log \mid (x + 1) + \sqrt{x^2 + 2x + 5} \mid + C$

Answer: C

Solution:

$$\text{Let} \quad I = \int \sqrt{5 - 2x + x^2} dx$$

$$I = \int \sqrt{(x-1)^2 + (2)^2}$$

We know that

$$\left[\because \int \sqrt{x^2+a^2} = \frac{x}{2}\sqrt{x^2+a^2} + \frac{a^2}{2}\log\left|x+\sqrt{x^2+a^2}\right|\right]$$

$$= \frac{x-1}{2}\sqrt{5-2x+x^2} + 2\log\left|(x-1)+\sqrt{5-2x+x^2}\right| + C$$

Question8

$\int \frac{1}{1+3\sin^2 x+8\cos^2 x} dx$ **is equals to**

KCET 2023

Options:

A. $\tan^{-1}\left(\frac{2\tan x}{3}\right)+C$

B. $\frac{1}{6}\tan^{-1}\left(\frac{2\tan x}{3}\right)+C$

C. $6\tan^{-1}\left(\frac{2\tan x}{3}\right)+C$

D. $\frac{1}{6}\tan^{-1}(2\tan x)+C$

Answer: B

Solution:

Let $I = \int \frac{1}{1+3\sin^2 x+8\cos^2 x} dx$

Dividing the numerator and denominator by $\cos^2 x$, we get

$$\Rightarrow I = \int \frac{\sec^2 x}{\sec^2 x + 3\tan^2 x + 8} dx$$
$$\Rightarrow I = \int \frac{\sec^2 x}{1+\tan^2 x + 3\tan^2 x + 8} dx$$
$$\Rightarrow I = \int \frac{\sec^2 x}{4\tan^2 x + 9} dx$$

Putting, $\tan x = t \Rightarrow \sec^2 x dx = dt$, we get

$$I=\int \frac{dt}{4t^2+9}=\frac{1}{4}\int \frac{dt}{t^2+\left(\frac{3}{2}\right)^2}+C$$
$$I=\frac{1}{4}\times\frac{1}{3/2}\tan^{-1}\left(\frac{t}{3/2}\right)+C$$
$$\Rightarrow \quad I=\frac{1}{6}\tan^{-1}\left(\frac{2t}{3}\right)+C=\frac{1}{6}\tan^{-1}\left(\frac{2\tan x}{3}\right)+C$$

Question9

$\int \frac{\cos 2x-\cos 2\alpha}{\cos x-\cos \alpha}dx$ **is equal to**

KCET 2022

Options:

A. $2(\sin x-x\cos\alpha)+c$

B. $2(\sin x+x\cos\alpha)+c$

C. $2(\sin x-2x\cos\alpha)+c$

D. $2(\sin x+2x\cos\alpha)+c$

Answer: B

Solution:

$$\text{Let } I=\int \frac{\cos 2x-\cos 2\alpha}{\cos x-\cos \alpha}dx$$
$$I=\int \frac{\left(2\cos^2 x-1\right)-\left(2\cos^2\alpha-1\right)}{\cos x-\cos\alpha}dx$$
$$I=2\int \frac{\cos^2 x-\cos^2\alpha}{\cos x-\cos\alpha}dx$$
$$=2\int \frac{(\cos x-\cos\alpha)(\cos x+\cos\alpha)}{(\cos x-\cos\alpha)}dx$$
$$=2\int(\cos x+\cos\alpha)dx=2(\sin x+x\cos\alpha)+c$$

Question10

If $\int \frac{dx}{(x+2)(x^2+1)} = a \log\left|1+x^2\right| + b\tan^{-1}x + \frac{1}{5}\log|x+2| + c$, then

KCET 2022

Options:

A. $a = \frac{-1}{10}, b = \frac{2}{5}$

B. $a = \frac{1}{10}, b = \frac{2}{5}$

C. $a = \frac{-1}{10}, b = \frac{-2}{5}$

D. $a = \frac{1}{10}, b = \frac{-2}{5}$

Answer: A

Solution:

Given,

$\int \frac{dx}{(x+2)(x^2+1)} = a\log|1+x^2| + b\tan^{-1}x + \frac{1}{5}\log|x+2| + c$ .... (i)

$$\text{Let } \frac{1}{(x+2)(x^2+1)} = \frac{A}{(x+2)} + \frac{Bx+C}{(x^2+1)}$$
$$\therefore \quad 1 = A\left(x^2+1\right) + (x+2)(Bx+C)$$
$$1 = Ax^2 + A + Bx^2 + Cx + 2Bx + 2C$$
$$1 = (A+B)x^2 + x(2B+C) + A + 2C$$

Here, $A+B=0$,

$2B+C=0$ and $A+2C=1$

On solving them, we get

$A = \frac{1}{5}, B = \frac{-1}{5}$ and $C = \frac{2}{5}$

Therefore, $\int \frac{dx}{(x+2)(x^2+1)}$

$$= \int\left(\frac{1}{5(x+2)} + \frac{\frac{-1}{5}x + \frac{2}{5}}{x^2+1}\right)$$
$$= \frac{1}{5}\int \frac{1}{(x+2)}dx - \frac{1}{5}\int \frac{x}{x^2+1}dx + \frac{2}{5}\int \frac{dx}{1+x^2}$$
$$= \frac{1}{5}\log|x+2| - \frac{1}{10}\log\left|1+x^2\right| + \frac{2}{5}\tan^{-1}x + c$$
$$= -\frac{1}{10}\log\left(1+x^2\right) + \frac{2}{5}\tan^{-1}x + \frac{1}{5}\log|x+2| + c \quad \text{.... (ii)}$$

On comparing Eqs. (i) and (ii), we get

$a = \frac{-1}{10}$ and $b = \frac{2}{5}$

Question11

$\int \frac{x^3 \sin(\tan^{-1}(x^4))}{1+x^8} dx$ **is equal to**

KCET 2021

Options:

A. $\frac{-\cos(\tan^{-1}(x^4))}{4} + C$

B. $\frac{\cos(\tan^{-1}(x^4))}{4} + C$

C. $\frac{-\cos(\tan^{-1}(x^3))}{3} + C$

D. $\frac{\sin(\tan^{-1}(x^4))}{4} + C$

Answer: A

Solution:

Let $I = \int \frac{x^3 \sin[\tan^{-1}(x^4)]}{1+x^8}$

Let $\tan^{-1}(x^4) = t$.

On differentiating w.r.t. t, we get

$$\frac{1}{1+x^8} \times 4x^3 dx = dt$$

$$\Rightarrow \quad \frac{x^3}{1+x^8} dx = \frac{1}{4} dt$$

$$I = \frac{1}{4} \int \sin t dt$$

$$= -\frac{1}{4} \cos t + C$$

$$= -\frac{1}{4} \cos(\tan^{-1}(x^4)) + C$$

Question12

The value of $\int \frac{x^2 dx}{\sqrt{x^6+a^6}}$ is equal to

KCET 2021

Options:

A. $\log\left|x^3+\sqrt{x^6+a^6}\right|+C$

B. $\log\left|x^3-\sqrt{x^6+a^6}\right|+C$

C. $\frac{1}{3}\log\left|x^3+\sqrt{x^6+a^6}\right|+C$

D. $\frac{1}{3}\log\left|x^3-\sqrt{x^6+a^6}\right|+C$

Answer: C

Solution:

Let $I=\int \frac{x^2}{\sqrt{x^6+a^6}} dx$

Let $x^3=t$

$$\Rightarrow \quad 3x^2 dx = dt$$

$$I=\frac{1}{3}\int \frac{1}{\sqrt{t^2+\left(a^3\right)^2}} dt$$

$$=\frac{1}{3}\log\left|t+\sqrt{t^2+a^6}\right|+C$$

$$=\frac{1}{3}\log\left|x^3+\sqrt{x^6+a^6}\right|+C$$

Question13

The value of $\int \frac{x e^x dx}{(1+x)^2}$ is equal to

KCET 2021

Options:

A. $e^x(1+x)+C$

B. $e^x\left(1+x^2\right)+C$

C. $e^x(1+x)^2+C$

D. $\frac{e^x}{1+x}+C$

Answer: D

Solution:

$$\begin{aligned}\text{Let } I &= \int \frac{xe^x dx}{(1+x)^2} \\ &= \int \frac{e^x(x+1-1)}{(1+x)^2} dx \\ &= \int e^x\left[\frac{1}{1+x}+\left(\frac{-1}{(1+x)^2}\right)\right] dx\end{aligned}$$

Let $\frac{1}{1+x}=f(x)$

$\therefore \quad f'(x)=-\frac{1}{(1+x)^2}$

Using the formula, $\int e^x\left[f(x)+f'(x)\right] dx$

$$=e^x f(x)+C$$

$$\Rightarrow \quad I=e^x\left(\frac{1}{1+x}\right)+C=\frac{e^x}{1+x}+C$$

Question14

The value of $\int e^x\left[\frac{1+\sin x}{1+\cos x}\right] dx$ is equal to

KCET 2021

Options:

A. $e^x \tan \frac{x}{2}+C$

B. $e^x \tan x+C$

C. $e^x(1+\cos x)+C$

D. $e^x(1+\sin x)+C$

Answer: A

Solution:

$$\text{Let } I = \int e^x \left(\frac{1 + \sin x}{1 + \cos x} \right) dx$$
$$= \int e^x \left(\frac{1 + 2 \sin \frac{x}{2} \cos \frac{x}{2}}{2 \cos^2 \frac{x}{2}} \right) dx$$
$$= \int e^x \left[\frac{\sec^2 \frac{x}{2}}{2} + \tan \frac{x}{2} \right] dx$$

$$\text{Let} \quad f(x) = \tan \frac{x}{2}$$
$$\therefore \quad f'(x) = \frac{\sec^2 \frac{x}{2}}{2}$$

Using the formula, $\int e^x [f(x) + f'(x)] dx$

$$= e^x f(x) + C$$
$$\Rightarrow \quad I = e^x \times \tan \frac{x}{2} + C$$

Question15

The value of $\int \frac{1+x^4}{1+x^6} dx$ is

KCET 2020

Options:

A. $\tan^{-1} x + \tan^{-1} x^3 + C$

B. $\tan^{-1} x + \frac{1}{3} \tan^{-1} x^3 + C$

C. $\tan^{-1} x - \frac{1}{3} \tan^{-1} x^3 + C$

D. $\tan^{-1} x + \frac{1}{3} \tan^{-1} x^2 + C$

Answer: B

Solution:

We have, $\int \frac{1+x^4}{1+x^6} dx$

$$= \int \frac{1+x^4}{1+x^6} \times \frac{1+x^2}{1+x^2} dx$$
$$= \int \frac{1+x^2+x^4+x^6}{(1+x^6)} dx$$
$$= \int \frac{1+x^6}{(1+x^6)1+x^2} dx + \int \frac{x^2(1+x^2)}{(1+x^6)(1+x^2)} dx$$
$$= \int \frac{dx}{1+x^2} + \int \frac{x^2}{1+x^6} dx$$
$$= \tan^{-1} x + \int \frac{x^2}{1+(x^3)^2} dx$$
$$= \tan^{-1} x + \int \frac{dt/3}{1+t^2}$$

Put $x^3 = t$

$$\Rightarrow \quad 3x^2 dx = dt$$
$$\Rightarrow \quad x^2 dx = \frac{dt}{3}$$
$$= \tan^{-1} x + \frac{1}{3} \tan^{-1}(t) + C$$
$$= \tan^{-1} x + \frac{1}{3} \tan^{-1}(x^3) + C$$

Question16

The value of $\int e^{\sin x} \sin 2x dx$ is

KCET 2020

Options:

A. $2e^{\sin x}(\sin x - 1) + C$

B. $2e^{\sin x}(\sin x + 1) + C$

C. $2e^{\sin x}(\cos x + 1) + C$

D. $2e^{\sin x}(\cos x - 1) + C$

Answer: A

Solution:

We have, $\int e^{\sin x} \sin 2x dx$

$$= \int e^{\sin x} \cdot (2 \sin x \cos x) dx$$

$$= 2 \int e^{\sin x} \sin x \cos x dx$$

let $\sin x = t$

$$\Rightarrow \cos x dx = dt = 2 \int e^t \cdot t dt$$

$$= 2 \left[t \int e^t dt - \int \left(\frac{d}{dt}(t) \int e^t dt \right) dt \right]$$

$$= 2 \left[t \cdot e^t - \int e^t dt \right]$$

$$= 2 \left[t \cdot e^t - e^t \right] + C$$

$$= 2 \left[e^t (t-1) \right] + C$$

$$= 2 \left[e^{\sin x} (\sin x - 1) \right] + C$$

$$= 2 e^{\sin x} (\sin x - 1) + C$$

Question17

If $\int \frac{3x+1}{(x-1)(x-2)(x-3)} dx A \log|x-1| B \log|x-2| + C \log|x-3| + C$, then the values of A, B and C are respectively

KCET 2020

Options:

A. $5, -7, -5$

B. $2, -7, -5$

C. $5, -7, 5$

D. $2, -7, 5$

Answer: D

Solution:

We have,

$\int \frac{3x+1}{(x-1)(x-2)(x-3)} dx$

Let $\frac{3x+1}{(x-1)(x-2)(x-3)}=\frac{A}{x-1}+\frac{B}{x-2}+\frac{C}{x-3}$
$\Rightarrow \quad (3x+1)=A(x-2)(x-3)+B(x-1)(x-3)+C(x-1)(x-2)$
put $x-1=0$
$\Rightarrow \quad x=1$
Then, $3\times 1+1=A(-1)(-2)$
$\Rightarrow \quad 4=2A$
$\Rightarrow \quad A=2$
Put $\quad x-2=0$
$\Rightarrow \quad x=2$
Then, $7=B(2-1)(2-3)$
$\Rightarrow \quad 7=B(1)(-1)$
$\Rightarrow \quad B=-7$
And put $x-3=0$
$\Rightarrow \quad x=3$
Then, $10=C(3-1)(3-2)$
$\Rightarrow \quad 10=C(2)(1)$
$\Rightarrow \quad C=5$

$\therefore \frac{3x+1}{(x-1)(x-2)(x-3)}=\frac{2}{x-1}-\frac{7}{x-2}+\frac{5}{x-3}$
$\therefore \int \frac{3x+1}{(x-1)(x-2)(x-3)}dx$
$=\int \frac{2}{x-1}dx-\int \frac{7}{x-2}dx+\int \frac{5}{x-3}dx$
$\Rightarrow \int \frac{3x+1}{(x-1)(x-2)(x-3)}dx$
$=2\log|x-1|-7\log|x-2|+5\log|x-3|+C$
$=A\log|x-1|+B\log|x-2|+C\log|x-3|+C$ (Given)
$\Rightarrow A=2, B=-7, C=5$

Question18

$\int x^3 \sin 3x dx=$

KCET 2019

Options:

A. $-\frac{x^3 \cdot \cos 3x}{3}+\frac{x^2 \sin 3x}{3}+\frac{2x \cos 3x}{9}-\frac{2 \sin 3x}{27}+C$

B. $-\frac{x^3 \cdot \cos 3x}{3}-\frac{x^2 \sin 3x}{3}+\frac{2x \cos 3x}{9}-\frac{2 \sin 3x}{27}+C$

C. $-\frac{x^3 \cdot \cos 3x}{3}+\frac{x^2 \sin 3x}{3}-\frac{2x \cos 3x}{9}-\frac{2 \sin 3x}{27}+C$

D. $\frac{x^3 \cdot \cos 3x}{3}+\frac{x^2 \sin 3x}{3}-\frac{2x \cos 3x}{9}-\frac{2 \sin 3x}{27}+C$

Answer: A

Solution:

key Idea Use Integration by parts as

$$\int u \cdot v dx = u \int v dx - \int\left(\frac{d}{dx}(u) \int v dx\right) dx$$
$$\Rightarrow x^3 \int \sin 3x dx - \int\left(\frac{d}{dx}\left(x^3\right) \int \sin 3x dx\right) dx$$
$$\Rightarrow x^3\left(\frac{-\cos 3x}{3}\right)-\int\left(3x^2\right)\left(-\frac{\cos 3x}{3}\right) dx$$
$$\Rightarrow \frac{-x^3 \cos 3x}{3}+\int x^2 \cos 3x dx$$

$$\Rightarrow -\frac{x^3 \cos 3x}{3}+\left[\begin{array}{r} x^2 \int \cos 3x dx - \int\left(\frac{d}{dx}\left(x^2\right)\right) \\ \int \cos 3x dx \end{array}\right] dx$$
$$\Rightarrow \frac{-x^3 \cos 3x}{3}+\left[\frac{x^2 \sin 3x}{3}-\int\left(2x \cdot \frac{\sin 3x}{3}\right) dx\right]$$
$$\Rightarrow \frac{-x^2 \cos 3x}{3}+\frac{1}{3}\left[x^2 \sin 3x-2 \int x \sin 3x dx\right]$$

$$\Rightarrow \frac{-x^2 \cos 3x}{3}+\frac{1}{3}\left[x^2 \sin 3x-2\left\{x \int \sin 3x dx-\int\left(\frac{d}{dx}(x) \int \sin 3x dx\right) dx\right]\right.$$

$$\Rightarrow \frac{-x^2 \cos 3x}{3}+\frac{1}{3}\left[x^2 \sin 3x-2\left\{\begin{array}{l} x\left(\frac{-\cos 3x}{3}\right) \\ -\int\left(\frac{-\cos 3x}{3}\right) \end{array}\right\}\right]+C$$

$$\Rightarrow \frac{-x^2 \cos 3x}{3}+\frac{x^2 \sin 3x}{3}+\frac{2x \cos 3x}{9}-\frac{2 \sin 3x}{27}+C$$

Question19

$$\int \frac{1}{\sqrt{x}+x\sqrt{x}} dx=$$

KCET 2019

Options:

A. $\tan^{-1} \sqrt{x}+C$

B. $2 \log(\sqrt{x}+1)+C$

C. $2 \tan^{-1} \sqrt{x}+C$

D. $\frac{1}{2}\tan^{-1}\sqrt{x}+C$

Answer: C

Solution:

Let $I=\int \frac{dx}{\sqrt{x}(x+1)}$

put $\sqrt{x}=t \Rightarrow x=t^2 \Rightarrow dx=2tdt$

$$\begin{aligned}\therefore \quad I &= \int \frac{2tdt}{t\left(t^2+1\right)}=2\int \frac{dt}{t^2+1}=2\tan^{-1}t+c \\ &= 2\tan^{-1}\sqrt{x}+c \quad (\text{ since, } t=\sqrt{x})\end{aligned}$$

Question20

$$\int \frac{2x-1}{(x-1)(x+2)(x-3)}dx = A\log|x-1|+B\log|x+2|+C\log|x-3|+K$$

Then A, B, C are respectively

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Options:

A. $\frac{1}{6}, \frac{-1}{3}, \frac{1}{3}$

B. $\frac{-1}{6}, \frac{1}{3}, \frac{1}{3}$

C. $\frac{-1}{6}, \frac{-1}{3}, \frac{1}{2}$

D. $\frac{1}{6}, \frac{1}{3}, \frac{1}{5}$

Answer: C

Solution:

We have $\int \frac{2x-1}{(x-1)(x+2)(x-3)}dx$
$= A\log|x-1|+B\log|x+2|+C\log|x-3|+K$

Let, $\frac{2x-1}{(x-1)(x+2)(x-3)}=\frac{A}{x-1}+\frac{B}{x+2}+\frac{C}{x-3}$

$\Rightarrow 2x - 1 = A(x+2)(x-3) + B(x-1)$
$(x-3) + C(x-1)(x+2)$

put $x - 1 = 0 \Rightarrow x = 1$ then, $2 \times 1 - 1 = A(3)(-2)$

$\Rightarrow \quad A = -\frac{1}{6}$

put $x + 2 = 0 \Rightarrow x = -2$

then, $2(-2) - 1 = B(-2-1)(-2-3) \Rightarrow B = -\frac{1}{3}$

put $x - 3 = 0 \Rightarrow x = 3$

then, $2 \times 3 - 1 = c(3-1)(3-2) \Rightarrow c = \frac{1}{2}$

Question21

$\int \frac{1}{1+e^x} dx$ is equal to

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Options:

A. $\log_e \left(\frac{e^x+1}{e^x}\right) + C$

B. $\log_e \left(\frac{e^x-1}{e^x}\right) + C$

C. $\log_e \left(\frac{e^x}{e^x+1}\right) + C$

D. $\log_e \left(\frac{e^x}{e^x-1}\right) + C$

Answer: C

Solution:

Let's evaluate the integral:

$I = \int \frac{1}{1+e^x} dx$

First, rewrite the integrand:

$I = \int \frac{e^{-x}}{e^{-x}+1} dx$

Now, use the substitution method. Let:

$u = e^{-x} + 1$

Differentiating both sides with respect to x, we get:

$\frac{du}{dx} = -e^{-x} \quad \Rightarrow \quad -e^{-x} dx = du$

Substituting, the integral becomes:

$I = -\int \frac{1}{u} \, du$

This integral simplifies to:

$I = -\log |u| + C$

Substitute back $u = e^{-x} + 1$:

$I = -\log(e^{-x} + 1) + C$

Using the properties of logarithms:

$I = \log\left(\frac{1}{e^{-x}+1}\right) + C$

Rewriting $\frac{1}{e^{-x}+1}$ in terms of e^{x}:

$I = \log\left(\frac{e^{x}}{e^{x}+1}\right) + C$

So, the final answer is:

$I = \log\left(\frac{e^{x}}{e^{x}+1}\right) + C$

Question22

$\int \frac{1}{\sqrt{3-6x-9x^2}} dx$ is equal to

KCET 2018

Options:

A. $\sin^{-1}\left(\frac{3x+1}{2}\right) + C$

B. $\sin^{-1}\left(\frac{3x+1}{6}\right) + c$

C. $\frac{1}{3}\sin^{-1}\left(\frac{3x+1}{2}\right) + C$

D. $\sin^{-1}\left(\frac{2x+1}{3}\right) + C$

Answer: C

Solution:

We need to evaluate the integral:

$I = \int \frac{1}{\sqrt{3-6x-9x^2}} \, dx$

First, let's rewrite the expression under the square root:

$I = \int \frac{1}{\sqrt{3-(9x^2+6x+1)+1}}\,dx$

Notice that $9x^2 + 6x + 1$ can be rewritten to form a perfect square. We have:

$9x^2 + 6x + 1 = (3x + 1)^2$

Therefore, the integral becomes:

$I = \int \frac{1}{\sqrt{(2)^2-(3x+1)^2}}\,dx$

Next, let's perform a substitution. Let:

$3x + 1 = t$

Then the differential becomes:

$d(3x + 1) = dt \quad \Longrightarrow \quad 3\,dx = dt \quad \Rightarrow \quad dx = \frac{1}{3}\,dt$

Substituting these into the integral, we get:

$I = \frac{1}{3} \int \frac{1}{\sqrt{(2)^2-t^2}}\,dt$

The integral $\int \frac{1}{\sqrt{a^2-t^2}}\,dt$ is recognized as the inverse sine function:

$\int \frac{1}{\sqrt{a^2-t^2}}\,dt = \sin^{-1}\left(\frac{t}{a}\right) + C$

Applying this formula with $a = 2$, we have:

$I = \frac{1}{3}\sin^{-1}\left(\frac{t}{2}\right) + C$

Substitute back $t = 3x + 1$:

$I = \frac{1}{3}\sin^{-1}\left(\frac{3x+1}{2}\right) + C$

Question23

$\int e^{\sin x} \cdot \left(\frac{\sin x+1}{\sec x}\right) dx$ is equal to

KCET 2018

Options:

A. $\sin x \cdot e^{\sin x} + C$

B. $\cos x \cdot e^{\sin x} + C$

C. $e^{\sin x} + C$

D. $e^{\sin x}(\sin x + 1) + C$

Answer: A

Solution:

To solve the integral $\int e^{\sin x} \cdot \left(\frac{\sin x+1}{\sec x}\right) dx$, let's start with the given function:

$I = \int e^{\sin x} \cdot \left(\frac{\sin x+1}{\sec x}\right) dx$

This can be rewritten as:

$I = \int e^{\sin x} \cdot (\sin x + 1) \cos x \, dx$

To simplify, we perform a substitution. Let $\sin x = t$. Then, the derivative is:

$\cos x \, dx = dt$

Substituting these into the integral, we get:

$I = \int e^t (t + 1) \, dt$

We recognize that this integral can be solved using the formula:

$\int e^x \{f(x) + f'(x)\} \, dx = e^x f(x) + C$

Applying this formula with $f(x) = t$ and $f'(x) = 1$, we have:

$I = te^t + C$

Substituting back $t = \sin x$, we arrive at the solution:

$I = \sin x \cdot e^{\sin x} + C$

Question24

$\int \sqrt{x^2 + 2x + 5} dx$ is equal to

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Options:

A.

$\frac{1}{2}(x+1)\sqrt{x^2+2x+5} + 2\log|x+1+\sqrt{x^2+2x+5}| + C$

B.

$(x+1)\sqrt{x^2+2x+5} + \frac{1}{2}\log\left|x+1+\sqrt{x^2+2x+5}\right| + C$

C.

$(x+1)\sqrt{x^2+2x+5} + 2\log|x+1+\sqrt{x^2+2x+5}| + C$

D.

$(x+1)\sqrt{x^2+2x+5} - 2\log|x+1+\sqrt{x^2+2x+5}+C$

Answer: A

Solution:

$$\text{Let } I = \int \sqrt{x^2+2x+5}dx$$
$$= \int \sqrt{(x+1)^2+4dx}$$
$$= \int \sqrt{(x+1)^2+(2)^2}dx$$
$$= \frac{x+1}{2}\sqrt{(x+1)^2+(2)^2} + \frac{(2)^2}{2}\log|x+1+\sqrt{(x+1)^2+(2)^2)}+c$$
$$= \frac{x+1}{2}\sqrt{x^2+2x+5}+2\log\left|x+1+\sqrt{x^2+2x+5}\right|+c$$

Question25

$\int \frac{(x+3)e^x}{(x+4)^2}dx$ **is equal to**

KCET 2017

Options:

A. $\frac{e^x}{(x+4)^2}+C$

B. $\frac{e^x}{(x+3)}+C$

C. $\frac{1}{(x+4)^2}+C$

D. $\frac{e^x}{(x+4)}+C$

Answer: D

Solution:

$$
\begin{aligned}
\text{Let } I &= \int \frac{x+3}{(x+4)^2} e^x dx \\
&= \int e^x \left[\frac{x+4-1}{(x+4)^2}\right] dx \\
&= \int e^x \left[\frac{1}{x+4} + \frac{-1}{(x+4)^2}\right] dx \\
&= \int e^x \frac{1}{x+4} dx + \int e^x \left(\frac{-1}{(x+4)^2}\right) dx \\
&= \frac{1}{X+4} e^x - \int e^x \cdot \left(\frac{-1}{(x+4)^2}\right) dx + \int e^x \left(-\frac{1}{(x+4)^2}\right) dx \\
&= \frac{e^x}{x+4} + C
\end{aligned}
$$

Question26

$\int \frac{\cos 2x - \cos 2\theta}{\cos x - \cos \theta} dx$ is equal to

KCET 2017

Options:

A. $2(\sin x + x \cos \theta) + C.$

B. $2(\sin x - x \cos \theta) + C$

C. $2(\sin x + 2x \cos \theta) + C$

D. $2(\sin x - 2x \cos \theta) + C$

Answer: A

Solution:

$$
\begin{aligned}
Let\ I &= \int \frac{\cos 2x - \cos 2\theta}{\cos x - \cos \theta} dx \\
&= \int \frac{(2\cos^2 x - 1) - (2\cos^2 \theta - 1)}{\cos x - \cos \theta} dx \\
&= 2 \int \frac{\cos^2 x - \cos^2 \theta}{\cos x - \cos \theta} dx \\
&= 2 \int \frac{(\cos x - \cos \theta)(\cos x + \cos \theta)}{\cos x - \cos \theta} dx \\
&= 2 \int (\cos x + \cos \theta) dx \\
&= 2[\sin x + x \cos \theta] + C
\end{aligned}
$$
